$$\lim_{x \to \infty} \left(\sqrt{x^2 + x} - \sqrt{x^2 + 4} \right)$$

$$\lim_{x \to \infty} \left(\sqrt{x^2 + x} - \sqrt{x^2 + 4} \right)$$

$$\lim_{x \to \infty} \left(x \sqrt{1 + \frac{1}{x}} - x \sqrt{1 + \frac{4}{x^2}} \right)$$

$$\lim_{x \to \infty} x \left(\sqrt{1 + \frac{1}{x}} - \sqrt{1 + \frac{4}{x^2}} \right)$$

$$\lim_{x \to \infty} \left(\sqrt{x^2 + x} - \sqrt{x^2 + 4} \right) \frac{\sqrt{x^2 + x} + \sqrt{x^2 + 4}}{\sqrt{x^2 + x} + \sqrt{x^2 + 4}}$$

$$\lim_{x \to \infty} \left(\frac{x^2 + x - x^2 - 4}{\sqrt{x^2 + x} + \sqrt{x^2 + 4}} \right)$$

$$\lim_{x \to \infty} \left(\frac{x\left(1 - \frac{4}{x}\right)}{x\left(\sqrt{1 + \frac{1}{x}} + \sqrt{1 + \frac{4}{x^2}} \right)} \right)$$

$$\lim_{x \to \infty} \left(\frac{\left(1 - \frac{4}{x}\right)}{\left(\sqrt{1 + \frac{1}{x}} + \sqrt{1 + \frac{4}{x^2}} \right)} \right)$$

$$\infty \cdot \left(\sqrt{1} - \sqrt{1} \right) = \frac{1 - 0}{\sqrt{1} + \sqrt{1}}$$

$$\infty \cdot 0 = \frac{1}{2}$$

Obviously ...